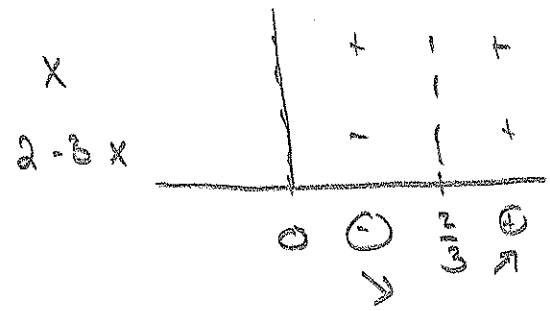
Math 4 Unit 7 optimization - Day 2

① $D = x^2 - x^3$

$D' = 2x - 3x^2$

$= x(2-3x)$



max difference when $x = \frac{2}{3}$

② $P = x \cdot y^2$ $\quad x + y = 20$

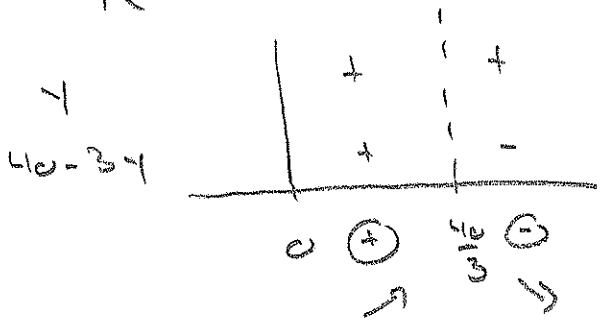
$x = 20 - y$

$P = (20-y)y^2$

$P = 20y^2 - y^3$

$\frac{dP}{dy} = 40y - 3y^2$

$= y(40-3y)$



max Product when

$y = \frac{40}{3}$, $x = 20 - \frac{40}{3}$

$= \frac{20}{3}$

③ $S = x + y$ $\quad xy = 64$

$y = \frac{64}{x}$

$S(x) = x + \frac{64}{x}$

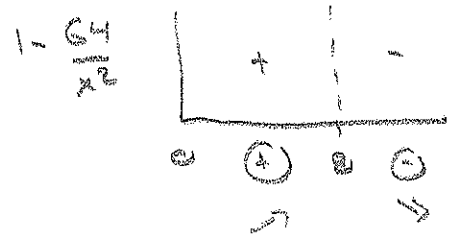
$S(x) = x + 64x^{-1}$

$S'(x) = 1 - 64x^{-2} = 1 - \frac{64}{x^2}$

$1 - \frac{64}{x^2} = 0$

$x^2 = 64$

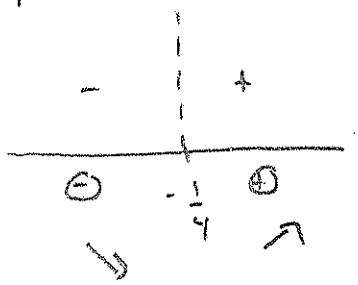
$x = 8$



max sum when $x = 8$, $y = 8$

④ $S = 2x^2 + x$

$S' = 4x + 1$



min sum when $x = -\frac{1}{4}$

⑤ $P = xy$ $\quad x - y = 22$

$x = y + 22$

$P = (y+22)y$

$P(y) = y^2 + 22y$

$P' = 2y + 22$

$= 2(y+11)$

	$-\infty$		-11		∞
2	+	|	+		
$y+11$	−	|	+		
	⊖	↘	−11	↗	⊕

min product when

$y = -11, \; x = 11$

⑥ $S = x^3 - 3x^2$

$S' = 3x^2 - 6x$

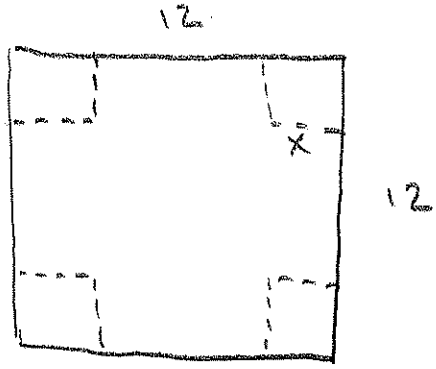
$= 3x(x-2)$

	$-\infty$		0		2		∞
$3x$	−	|	+		+		
$x-2$	−	|	−		+		
	⊕	↘	0	↘	2	↗	⊕

min difference when

$x = 2$

⑦



$V(x) = x(12-2x)(12-2x)$

$= x(144 - 48x + 4x^2)$

$= 4x^3 - 48x^2 + 144x$

$V'(x) = 12x^2 - 96x + 144$

$= 12(x^2 - 8x + 12)$

$= 12(x-6)(x-2)$

	$-\infty$		2		6		∞
12	+	|	+		+		
$x-6$	−	|	−		+		
$x-2$	−	|	+		+		
	⊕	↗	2	↘	⊖	↘	⊕

max volume when $x = 2$

dimensions $2 \times 8 \times 8$